Mathematics SL formula booklet

For use during the course and in the examinations

First examinations 2014

Contents

Prior learning

Area of a parallelogram	$A = b \times h$
Area of a triangle	$A = \dfrac{1}{2}(b \times h)$
Area of a trapezium	$A = \dfrac{1}{2}(a + b)h$
Area of a circle	$A = \pi r^2$
Circumference of a circle	$C = 2\pi r$
Volume of a pyramid	$V = \dfrac{1}{3}(\text{area of base} \times \text{vertical height})$
Volume of a cuboid (rectangular prism)	$V = l \times w \times h$
Volume of a cylinder	$V = \pi r^2 h$
Area of the curved surface of a cylinder	$A = 2\pi rh$
Volume of a sphere	$V = \dfrac{4}{3}\pi r^3$
Volume of a cone	$V = \dfrac{1}{3}\pi r^2 h$
Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2)	$\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}, \dfrac{z_1 + z_2}{2} \right)$

Topic 1—Algebra

1.1	The nth term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \dfrac{n}{2}(2u_1 + (n-1)d) = \dfrac{n}{2}(u_1 + u_n)$
	The nth term of a geometric sequence	$u_n = u_1 r^{n-1}$
	The sum of n terms of a finite geometric sequence	$S_n = \dfrac{u_1(r^n - 1)}{r - 1} = \dfrac{u_1(1 - r^n)}{1 - r}$, $r \neq 1$
	The sum of an infinite geometric sequence	$S_\infty = \dfrac{u_1}{1 - r}$, $\lvert r \rvert < 1$
1.2	Exponents and logarithms	$a^x = b \iff x = \log_a b$
	Laws of logarithms	$\log_c a + \log_c b = \log_c ab$
		$\log_c a - \log_c b = \log_c \dfrac{a}{b}$
		$\log_c a^r = r \log_c a$
	Change of base	$\log_b a = \dfrac{\log_c a}{\log_c b}$
1.3	Binomial coefficient	$\dbinom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$
	Binomial theorem	$(a+b)^n = a^n + \dbinom{n}{1}a^{n-1}b + \ldots + \dbinom{n}{r}a^{n-r}b^r + \ldots + b^n$

Topic 2—Functions and equations

2.4	Axis of symmetry of graph of a quadratic function	$f(x) = ax^2 + bx + c \implies$ axis of symmetry $x = -\dfrac{b}{2a}$
2.6	Relationships between logarithmic and exponential functions	$a^x = e^{x \ln a}$ $\log_a a^x = x = a^{\log_a x}$
2.7	Solutions of a quadratic equation	$ax^2 + bx + c = 0 \implies x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \ a \neq 0$
	Discriminant	$\Delta = b^2 - 4ac$

Topic 3—Circular functions and trigonometry

3.1	Length of an arc	$l = \theta r$
	Area of a sector	$A = \dfrac{1}{2}\theta r^2$
3.2	Trigonometric identity	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
3.3	Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
	Double angle formulae	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
3.6	Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C \,;\ \cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$
	Sine rule	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
	Area of a triangle	$A = \dfrac{1}{2}ab\sin C$

Topic 4—Vectors

4.1	Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{v_1{}^2 + v_2{}^2 + v_3{}^2}$
4.2	Scalar product	$\boldsymbol{v} \cdot \boldsymbol{w} = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \cos\theta$ $\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$
	Angle between two vectors	$\cos\theta = \dfrac{\boldsymbol{v} \cdot \boldsymbol{w}}{\lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert}$
4.3	Vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

Topic 5—Statistics and probability

5.2	Mean of a set of data	$\bar{x} = \dfrac{\sum\limits_{i=1}^{n} f_i x_i}{\sum\limits_{i=1}^{n} f_i}$
5.5	Probability of an event A	$\mathrm{P}(A) = \dfrac{n(A)}{n(U)}$
	Complementary events	$\mathrm{P}(A) + \mathrm{P}(A') = 1$
5.6	Combined events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
	Mutually exclusive events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
	Conditional probability	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B \mid A)$
	Independent events	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$
5.7	Expected value of a discrete random variable X	$\mathrm{E}(X) = \mu = \sum\limits_{x} x\,\mathrm{P}(X = x)$
5.8	Binomial distribution	$X \sim \mathrm{B}(n, p) \;\Rightarrow\; \mathrm{P}(X = r) = \binom{n}{r} p^r (1-p)^{n-r}, \; r = 0, 1, \ldots, n$
	Mean	$\mathrm{E}(X) = np$
	Variance	$\mathrm{Var}(X) = np(1-p)$
5.9	Standardized normal variable	$z = \dfrac{x - \mu}{\sigma}$

Topic 6—Calculus

6.1	Derivative of $f(x)$	$y = f(x) \implies \dfrac{dy}{dx} = f'(x) = \lim\limits_{h \to 0}\left(\dfrac{f(x+h) - f(x)}{h}\right)$
6.2	Derivative of x^n	$f(x) = x^n \implies f'(x) = nx^{n-1}$
	Derivative of $\sin x$	$f(x) = \sin x \implies f'(x) = \cos x$
	Derivative of $\cos x$	$f(x) = \cos x \implies f'(x) = -\sin x$
	Derivative of $\tan x$	$f(x) = \tan x \implies f'(x) = \dfrac{1}{\cos^2 x}$
	Derivative of e^x	$f(x) = e^x \implies f'(x) = e^x$
	Derivative of $\ln x$	$f(x) = \ln x \implies f'(x) = \dfrac{1}{x}$
	Chain rule	$y = g(u),\ u = f(x) \implies \dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
	Product rule	$y = uv \implies \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
	Quotient rule	$y = \dfrac{u}{v} \implies \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
6.4	Standard integrals	$\displaystyle\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C,\quad n \neq -1$
		$\displaystyle\int \dfrac{1}{x}\,dx = \ln x + C,\ x > 0$
		$\displaystyle\int \sin x\,dx = -\cos x + C$
		$\displaystyle\int \cos x\,dx = \sin x + C$
		$\displaystyle\int e^x\,dx = e^x + C$
6.5	Area under a curve between $x = a$ and $x = b$	$A = \displaystyle\int_a^b y\,dx$
	Volume of revolution about the x-axis from $x = a$ to $x = b$	$V = \displaystyle\int_a^b \pi y^2\,dx$
6.6	Total distance travelled from t_1 to t_2	$\text{distance} = \displaystyle\int_{t_1}^{t_2} |v(t)|\,dt$